

Hong Kong Mathematics Olympiad (1996 – 97)

Final Event 1 (Individual)

香港數學競賽 (1996 – 97)

決賽項目 1 (個人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) Given that $\frac{3}{a} + \frac{1}{u} = \frac{7}{2}$ and $\frac{2}{a} - \frac{3}{u} = 6$ are simultaneous equations in a and u .

$a =$

Solve for a .

已知 $\frac{3}{a} + \frac{1}{u} = \frac{7}{2}$ 及 $\frac{2}{a} - \frac{3}{u} = 6$ 為 a 與 u 的聯立方程。求 a 的解。

- (ii) Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b .

$b =$

方程 $px + qy + bz = 1$ 的根分別為 $(0, 3a, 1)$ 、 $(9a, -1, 2)$ 和 $(0, 3a, 0)$ 。求係數 b 的值。

- (iii) Find c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(-2, 2)$.

$c =$

若 $y = mx + c$ 的圖像經過 $(b + 4, 5)$ 及 $(-2, 2)$ 兩點。求 c 。

- (iv) The solution of the inequality $x^2 + 5x - 2c \leq 0$ is $d \leq x \leq 1$. Find d .

$d =$

不等式 $x^2 + 5x - 2c \leq 0$ 的解為 $d \leq x \leq 1$ 。求 d 。

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Final Event 2 (Individual)

香港數學競賽 (1996 – 97)

決賽項目 2 (個人)

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除非特別聲明，答案須用數字表達，並化至最簡。

(i) By considering :

$$\frac{1^2}{1} = 1, \frac{1^2 + 2^2}{1 + 2} = \frac{5}{3}, \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3} = \frac{7}{3}, \frac{1^2 + 2^2 + 3^2 + 4^2}{1 + 2 + 3 + 4} = 3, \text{ find } a \text{ such that}$$

$$\frac{1^2 + 2^2 + \cdots + a^2}{1 + 2 + \cdots + a} = \frac{25}{3}.$$

考慮：

$$\frac{1^2}{1} = 1, \frac{1^2 + 2^2}{1 + 2} = \frac{5}{3}, \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3} = \frac{7}{3}, \frac{1^2 + 2^2 + 3^2 + 4^2}{1 + 2 + 3 + 4} = 3, \text{ 求 } a \text{ 使得}$$

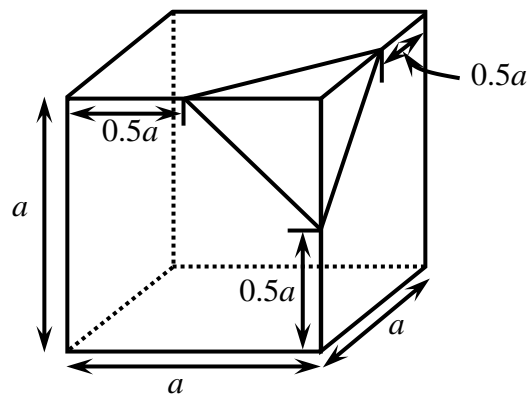
$$\frac{1^2 + 2^2 + \cdots + a^2}{1 + 2 + \cdots + a} = \frac{25}{3}.$$

$a =$

(ii) A triangular pyramid is cut from a corner of a cube with side length a cm as the figure shown. If the volume of the pyramid is $b \text{ cm}^3$, find b .

$b =$

如圖所示，從正立方體的一角割出一個三角椎體。若三角椎體的體積為 $b \text{ cm}^3$ ，求 b 。



- (iii) If the value of $x^2 + cx + b$ is not less than 0 for all real number x , find the maximum value of c .

$c =$

若對於所有實數 x ， $x^2 + cx + b$ 不小於 0，求 c 的最大值。

- (iv) If the unit digit of 1997^{1997} is $c - d$, find d .

$d =$

若 1997^{1997} 的個位數為 $c - d$ ，求 d 。

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Final Event 3 (Individual)

香港數學競賽 (1996 – 97)

決賽項目 3 (個人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) The average of a , b , c and d is 8. If the average of a , b , c , d and P is P , find P .

$P =$

a 、 b 、 c 和 d 的平均值為 8。若 a 、 b 、 c 、 d 和 P 的平均值為 P ，求 P 。

- (ii) If the lines $2x+3y+2=0$ and $Px+Qy+3=0$ are parallel, find Q .

$Q =$

若直線 $2x+3y+2=0$ 和 $Px+Qy+3=0$ 互相平行，求 Q 。

- (iii) The perimeter and the area of an equilateral triangle are Q cm and $\sqrt{3}R$ cm² respectively. Find R .

$R =$

若等邊三角形的周界和面積分別為 Q cm 和 $\sqrt{3}R$ cm²。求 R 。

- (iv) If $(1+2+\cdots+R)^2=1^2+2^2+\cdots+R^2+S$, find S .

$S =$

若 $(1+2+\cdots+R)^2=1^2+2^2+\cdots+R^2+S$ ，求 S 。

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Final Event 4 (Individual)

香港數學競賽 (1996 – 97)

決賽項目 4 (個人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) If each interior angle of a n -sided regular polygon is 140° , find n .

$n =$

若正 n 邊形的內角為 140° ，求 n 。

- (ii) If the solution of the inequality $2x^2 - nx + 9 < 0$ is $k < x < b$, find b .

$b =$

若不等式 $2x^2 - nx + 9 < 0$ 的解為 $k < x < b$ ，求 b 。

- (iii) If $cx^3 - bx + x - 1$ is divided by $x + 1$, the remainder is -7 , find c .

$c =$

若 $cx^3 - bx + x - 1$ 除以 $x + 1$ ，餘數為 -7 ，求 c 。

- (iv) If $x + \frac{1}{x} = c$ and $x^2 + \frac{1}{x^2} = d$, find d .

$d =$

若 $x + \frac{1}{x} = c$ 和 $x^2 + \frac{1}{x^2} = d$ ，求 d 。

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Final Event 5 (Individual)

香港數學競賽 (1996 – 97)

決賽項目 5 (個人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) The volume of a hemisphere with diameter a cm is $18\pi \text{ cm}^3$, find a .

$a =$

一直徑為 a 的半球體的體積為 $18\pi \text{ cm}^3$ ，求 a 。

- (ii) If $\sin 10a^\circ = \cos(360^\circ - b^\circ)$ and $0 < b < 90$, find b .

$b =$

若 $\sin 10a^\circ = \cos(360^\circ - b^\circ)$ 和 $0 < b < 90$ ，求 b 。

- (iii) The triangle is formed by the x -axis and y -axis and the line $bx + 2by = 120$. If the bounded area of the triangle is c , find c .

$c =$

一三角形是由 x -軸、 y -軸和直線 $bx + 2by = 120$ 所組成。若所包圍之三角形的面積為 c ，求 c 。

- (iv) If the difference of the two roots of the equation $x^2 - (c+2)x + (c+1) = 0$ is d , find d .

$d =$

若方程式 $x^2 - (c+2)x + (c+1) = 0$ 兩根之差為 d ，求 d 。